

## GRAPH-THEORETIC ANALYSIS FOR SUSTAINABLE URBAN STRUCTURE

Tamara mutaz, Zainab Khalid

### Abstract

The characterization of urban systems introduces several challenges regarding the complex structures of central business districts (CBDs) in cities. The traditional urban models are inadequate because they do not consider the non-linear networked nature of the city's economic and social dynamics. This results in an imbalance among the city's physical structures that correspond to the underlying processes taking place within it. Therefore, the main question shows a configurational nature: how can we methodically analyze the intrinsic spatial logic of the CBD to create a more effective urban design model? This paper presents a theoretical and methodological framework to examine this question by regarding Lower Manhattan as the principal case study of the analysis. The research aims to create an analytical model that fosters interactions within the CBD. The methodology employs graph-theoretic mathematical coding and spatial-syntax indices to examine the dynamics of the urban structure in the CBD. The indices included: vulnerability as a comprehensive attribute of urban network topology, Prüfer's code between two designated nodes, and three syntactic methodologies to analyze its spatial configuration. Moreover, the indices also include Q-analysis, Zipf's law, Markov chains, and fractal analysis which explain how shape, function, and change over time are related at various levels. The findings show that Lower Manhattan's CBD is complex, robust, and tends to have a planned network topology. In addition, there are correlations between the spatial integration indices and economic activity patterns while the grid remains an essential component of the economic functions. The analytical model presented by this study is predictive and quantitative because it activates the understanding of urban complexity. The findings show the connection between urban structure form, economic function, and the probability of exposure to potential vulnerabilities. The conclusions demonstrate how the proposed framework provides a model for urban designers to develop urban connectivity, detect the structural deficiencies, and employ solutions that promote sustainable, resilient, and economically viable urbanism.

**Keyword:** spatial analysis; graph theory; central business district; Lower Manhattan; urban structure.

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## 1. INTRODUCTION

Several non-linear factors affect the CBD effectiveness within the city. It is considered an essential component of the urban system because it represents a center of the economic influence (Lynch 1981; He et al. 2025). The conventional models frequently demonstrate inadequacy in articulating its complexity. Consequently, the return to the mathematical foundations in scientific research has increased to obtain more integrated methodologies for the analysis and modeling (Miller et al. 2023). On the other hand, graph theory has developed as a technique for representing the city as a network of connections. That is, a network of nodes and edges constituting the city's map (Batty 2013; Boeing 2019; Zhang et al. 2024). This aligns with the study's proposal of mathematical coding as a framework that employs the urban structure formal systems, algorithms, and computational logic to examine urban form and function (Kitchin and Perng 2016; Gomez and Lee 2025). This methodology develops the conventional urban structure theories, evolving them into a more generative and predictive science of urbanism (Patel and Roberts 2022). Previous literature defines two primary issues. The first is related to urban mathematics and topology. The objective is to demonstrate the formal connection between mathematical models and spatial configuration (Legendre 2011; Brelsford et al. 2015; Huang and Kim 2023). The second issue relates to the urban coding which investigates the application of these formal systems in urban design (Angelakis et al. 2017; Luque-Ayala et al. 2022; Singh and Martinez 2024). The literature also includes the principles extracted from the fractal geometry in terms of scaling and self-similarity (Batty and Longley 1994; Ali and Mustafa 2024; Chen and Wang 2023). Topology, on the other hand, is employed to analyze the urban connectivity (Tabak 2011; Rashid and Salman 2024; Novak and Chen 2025), while graph theory shows how cities are interconnected (Carlsson and Vejdemo-Johansson 2021; Fang, et al. 2024). However, despite the influence made by these works, the gap remains in our understanding of urban complexity. Moreover, current studies have not combined mathematical coding, using graph theory, into a comprehensive approach to study complexities of urban structure. The traditional approach for urban studies is inadequate to emphasize the complex topological complexity of urban structure. It offered an inadequate approach to solve problems associated with complex relationships between spatial interaction, economic activities, and human mobility (Ribeiro and Rybski 2023; Torres and Almeida 2025). This gap is represented by the inadequate use of multi-method approach to analyze this gap. This study introduces a new analytical approach to urban design science to go beyond the traditional theory and lacking to predict economic performance, to recognize urban

vulnerabilities to improve connectivity and resilience. This study hypothesizes that mathematical coding techniques using graph theory can analyze complexities associated with urban designs within the CBD. It predicts the economic performance of the CBD. The primary goal for this study is to assess new approach to analyze complexities associated with urban designs to use both mathematical coding techniques and concepts associated with economics of graphs for urban designs. The objectives are: (1) to explain mathematical coding and its correlation with urban design science; (2) to describe dominant concepts in urban design theory; (3) to examine the influence of graph-theoretic economic concepts on analyzing urban design complexities; and (4) to formulate a new approach for analyzing these complexities in urban design science. The proposed model will analyze essential configurational issues, such as the structural isomorphism, self-similarity, spatial organization, and connectivity. It will also relate these to economic behaviors. The case study analysis is applied to Lower Manhattan's CBD to foster integration of sustainable urbanization. After this introduction, the paper will proceed to a literature review, the case study description, materials and methods, the findings discussion, and finally the main conclusion of this study.

### Literature survey: Coding principles across disciplines

Coding concept is conventional in computation, it is considered a transdisciplinary process which is used in informatics organization, understanding, and a symbolic representation used for organizational clarity (McIntosh 2013; Cambridge Advanced Learner's Dictionary 2013). This concept is employed in urban settings using the rule-based systems. The difficulty remains in the understanding of the fundamental spatial logic regarding the complexities of urban structure. Coding in linguistics is defined as being governed by rules that are still fluid and highly context-dependent (Oller 2019). The linguistic Zipf elaborated a law that elucidates frequency distributions or word frequency related to brevity of use related to psychology (Zipf 2016). This concept is used in urban studies that compare the hierarchical organization of road networks within urban structures (e.g., Ribeiro & Rybski 2023). Similarly, shape grammar is originated in language and extended to design (Stiny 1980; Hosseini et al. 2024). Shape grammar provides an algorithmic basis for shape generation. However, it focuses on the rule-based generation rather than the complexities of human activities and culturally specific information embodied in spatial designs. In biology, coding is demonstrated through genetic coding and scaling laws which have largely influenced urban theory. The scaling laws including Kleiber's Law correlate body size to mammals' metabolic rates to create (measurements of infrastructure within the urban systems (Ribeiro and Diego, 2023; Isalgue,

Coch, and Serra, 2007; Pumain & Paulus, 2006; Rybski, Arcaute, and Batty, 2019). This tends to demonstrate limitations of urban systems which concern the resource allocation and opportunities within the urban settings. The Markov Chains, on the other hand, link coding and biology, defining coding by converting the (urban transformation) to set of probabilistic states (Norris 1997). These concepts facilitate the land use transition states (Li and Ning, 2023). Coding related to perception, focuses on the cognitive processes within sensory information gained by the environment (Tchertov 2019; Rogers 2017). This type of coding is used to convey the experience of reality is not just factual but also constructed by experience. Coding and cryptography are like computation, and they indicate a mathematical precision of information theory which underpins digital infrastructures (Shannon 1948; O'Regan 2020). Coding application through the cryptographic principles, including computational complexity, is used in the urban security systems (Yan 2013). It is distinctive in its capacity to provide control and accessibility within the urban landscape. This literature direction may exacerbate surveillance prevalence and intensify existing inequalities by prioritizing technological solutions over social and political factors. Coding in terms of complexity science, employs the mathematical abstractions to define urban networks; however, this method may show a potential for reductivism (Blanchard and Volchenkov 2008; Yu et al. 2022; Jazdzewska, 2022; Dawes and Ostwald, 2013; Boeing, 2018). Graph theory provides a comprehensive framework for delineating relationships within the urban system by representing dynamic social activities as interlinked nodes and edges. The objective is to ensure that this representation does not excessively simplify the complexities and variety of human interaction while game theory assumes rational agents and frequently irrational environments of urban areas (Nash 1950; Siegfried 2006). Emergence is a fundamental concept in complex systems theory (Portugali 2011), signifying that urban development arises from the interactions of simpler components. However, these models frequently depend on predetermined rules that render urban systems less stochastic and adaptable. Fractal geometry adeptly incorporates recursive patterns and self-similarity within urban morphology (e.g., Measuring global urban complexity from the aspect of living. Nevertheless, it frequently overlooks the social and political factors that influence these forms (Mandelbrot 1982; Batty & Longley 1994). The semi-lattice model similarly emphasizes interconnectedness yet may insufficiently account for the frictions and disconnections that are also significant aspects of urban life (Alexander 2017; Cannon 1984). The true challenge is to employ these potent analytical instruments while remaining aware of the city's social and spatial dynamics. See Table 1.

## Models of urban structure

The development of urban structure concept refers to the Chicago School's fundamental theories of industrial spatial organization (Heath et al. 2011). It demonstrates a persistent challenge in accurately representing the inherent relational characteristics of the city, this is developed by Alexander's thoughts. Alexander (1987) promoted a more generative viewpoint, by suggesting 'wholeness' of urban form and its development through dynamic processes. It also constitutes continuous growth and center formation or development (Alexander 1987). This approach treats urban structure not exclusively as physical but rather socially and economically significant too (Jiang and Yao 2012). This further supports our notion of defining and understanding space as social constructivism is actualized. Salingaros (2005) defined an "urban web" and determined its mathematical formulation (Salingaros 2005).

Schneider (2015) agrees with this idea by clarifying that these values become not imposed but rather inserted into its own physical character of space which results in an influence on its own structure (Schneider 2015). Current research has stimulated its network aspects emphasizing its socio-economic linkages. This emphasizes the overall global urban complexity (Burke 2021). Urban structure is therefore demonstrated to constitute both the physical and functional structure (Klopfer 2023). Perceptions and cognitive attributes pertain to subjective experience (objective) classification and structuring of spaces itself (Pacione 2009). This also stretches from monocentric to polycentric structures linked to the classic urban economy concepts (Pinho and Silva 2016).

The initial attempts to conceptualize Burgess concentric zones or sector concept initiated to depict urban industrial growth itself (Pacione, 2009). Rent theory was developed by Von Thünen used for market-centric surrounding land areas (Van Der Hoeven Frank et al. 2016). This theory clearly assesses economic valuations between land use or distance which was developed to assess its effectiveness (Agarwal et al., 2017). The Ricardian rent theory streamlines the varied and possibly disconnected functional configurations of spaces of urban cases (Roos et al. 2018). David Ricardo (Ricardian Theory) expanded rent theory to include land fertility and labor but not necessarily treated its intrinsic complexity in actual urban spaces itself being actual (Ricardo and Gonner 1891).

Alfred Weber's industry placement further emphasized to establish the optimal use of raw material, market, and labor using geometric relationships to optimize spatial efficiency (Weber and Friedrich 1929). Although influential during its era, it oversimplifies urban

**Table 1. Core Coding Principles Across Disciplines and Their Applications to Urban**

Discipline	Coding Principle	Definition	Critical Application to Urban Systems	Source
Linguistics	Zipf's Law (Power Law)	Statistical distribution of dominance, where brevity correlates with frequency of use.	Explains hierarchical urban patterns but risks reductionism by neglecting socio-political factors shaping those patterns.	(Zipf 2016)
Biology	Markov Chains	Stochastic model represents evolving phenomena over time.	Useful for modeling urban state transitions but abstracts complex social dynamics into probabilistic terms, risking oversimplification.	(Norris 1997)
	Scaling Laws (Kleiber's Law)	Relational proportions between components, i.e., metabolic scaling with body mass.	Reveals some city scaling patterns yet can obscure inequality and historical contingency in urban growth.	(Ribeiro and Diego 2023)
	Game Theory (Nash Equilibrium)	Computers expected payoffs among strategies, assuming rational actors.	This theory Models strategic urban interactions but assumes rationality/stability, often unrealistic in urban contexts.	(Siegfried 2006)
	Computational Difficulty	Based on difficulty factoring large numbers, key for cryptographic security.	Parallels urban system security but risks naturalizing power structures and ignores techno-social inequalities.	(Yan 2013)
Urban Networks	Fractal Geometry	Describes self-similar, recursive urban patterns.	Captures morphological complexity but eclipses underlying socio-economic and political forces.	(Cannon 1984; Al-Dabbagh A. A., & Ismail, K. J. A. 2025; Mahdi, M. S. & Hassan, A. 2016).
Urban Networks	Semi-Lattice Model	Network of overlapping urban connections.	Highlights relational order but may underplay urban conflict and fragmentation.	(Alexander 2017)
Underlying Framework	Graph Theory	Abstract system of nodes and edges modeling connections.	Powerful for mapping urban relations but reduces fluid social/spatial practices to static forms.	(O'Brien 2019; Yu et al. 2022)

phenomena to cost-reduction issues, which resulted in a neglect of the environmental and social elements associated with the spatial arrangement (Amiri et al. 2021). Christaller's central place theory proposed hierarchical settlement patterns, he proposed a hexagonal arrangement and hierarchical structure (Martí & Martínez-Gavara, 2023).

Contemporary theories, exemplified by Alonso's (1964) research, sought to associate commuting expenses with urban land prices, resulting in concentric models of rent gradients (Alonso 1964). His assumptions of uniformity and, importantly, monocentricity, although essential to urban economics, obscure multi-nodal urban configurations. Lowry (1964) incorporated land use, population, and employment into a regional framework, subsequently enhanced through entropy-maximization methods. He aimed at forecasting spatial distributions based on probabilistic principles (Wilson 2022).

Muth (1967) further underscored the notion of spatial harmony between residential zones and commuting through identifying the Central Business District as a main economic hub. This has been criticized for oversimplifying the intricate factors influencing urban expansion (Hewitt et al. 2022). Fujita et al. (2001) expanded bid-rent functions to include urban planning related to the city size. He addressed stratification and commuting effects. However, it is applied to residential contexts particularly.

Recent literature, which is based on the classic location theories, employed the structure and functional logic of urban and regional systems. For instance, Thünen's model is considered as a reference for contemporary research in terms of the geographical distribution of populations in relation to central nodes. Moreover, it is applied to the formulation of policies and allocation of forest land use (Roos et al. 2018). Another application of this model is shown in the understanding of urban land markets, basic structure, regulations, and the improvement of planting configuration geometry (Jäger & Kobayashi, 2020; Hao et al., 2025).

The Ricardian rent theory in recent literature is considered as one of the basic concepts of social progress. It is linked to population growth, which affects land rent growth (McDonald, 2018). At the same time, there have been efforts to introduce the theoretical rationale for institutional approaches to urban rent (Fair, 2024). Weber's industrial location theory is applied recently to computer science through iterative algorithms, finite sets of points in Euclidean spaces, and polynomial equation solving (Church et al. 2025). Furthermore, it is used to solve multi-use scenarios which have been developed through the application of an alternative location-allocation method (Amiri et al.

2021).

Recent discussions on this theory have evolved the idea of towniness into a broader concept of cityness, to describe the flow theory (Taylor and Hoyler, 2020). Other research has highlighted the diagrams that illustrate the governing scientific principles of the organization of periodic markets (Chao, 2020). Rent theory contributes to the development of the market concept and its critical significance for the distributional effects related to rental housing allocation (Fair, 2024). Contemporary research examines the geographies of rent burden and the urban-level determinants that significantly influence it (Samarin et al., 2024). Ira Lowry's model is distinctive for its basic algebraic equations which are developed to include intricate calculations while maintaining its fundamental framework through the principle of entropy maximization (Wilson 2022). This model also functions as a reference for comprehensive models such as the integration of intraregional commuting and consumption patterns (Gholami 2024).

Recent studies have employed this model to determine whether land-use or transportation strategies require regulation to achieve compaction in a traditional automobile-dependent urban environment (Chen and Wang 2025). Muth's model is developed to an approach that incorporates calibration rule sets (Hewitt et al., 2022). Notable advancements are evident in the ability of rational expectations models to define and evaluate market efficiency in financial economics (Delcey and Sergi 2022). The correlation between economic growth and population growth is increasingly complex. A catastrophic shock is believed to trigger the decline of lower-tier provincial cities (Fujita et al. 2021). Isard and Fujita model is employed by recent literature to examine port cities, concentrating on the interactions among individuals and economic equilibrium, along with the origins, evolution, and constraints of their geographical configurations (Coelho 2024). Although these studies employed mathematics to examine spatial and economic variables, they did not concentrate on the central business district or predict its performance. Although these models offer a mathematical framework for urban economics, they show challenges due to oversimplification compared to the spatial and social dynamics of the city.

#### Graph theory in urban design

Graph theory in urban studies shows abstracts the city into a network of nodes and edges. This mathematical representation is used to examine connectivity, inequality, and resilience (O'Brien, 2019). Spatial analysis, as developed by Hillier and Hanson (1984), supports the study of urban structural configuration. Space syntax theory relates the arrangement of



streets and spaces to patterns of human interaction. It demonstrates predictive capacity for pedestrian and vehicular movement by measuring integration, connectivity, and depth, thereby revealing the underlying spatial logic of the urban grid (Hillier and Iida 2005; Abbas and Al-Dujaili 2013). One of its key contributions is to the transformation from an abstract concept of permeability into a quantifiable metric (Hillier 1996).

This emphasis on configuration may be critiqued for neglecting the symbolic significances and cultural impediments that influence movement (Karimi 2012; Ratti 2004). Methodological reliance on axial maps has also raised concerns regarding reliability and objectivity (Dawes and Ostwald 2013). Subsequent developments, including angular segment analysis and visibility graph analysis directly enhances this model (Turner, 2007). The developed version of space syntax improved its representation of perceptual experience and predictive capability.

Network models are widely used in transportation studies, where graph-based algorithms such as Dijkstra and A\* are applied to route optimization (Dijkstra 1959; Nilsson et al. 1968). These approaches support multimodal planning and infrastructure resilience (Ceder 2016; Sun et al. 2024). However, efficiency-driven models often neglect experiential and embodied aspects of mobility, reinforcing socio-spatial disparities (Lucas 2012; Jensen 2009).

Graph-based morphological analysis examines urban tessellations and street grids by connecting the small-scale configurations to larger urban patterns (Fleischmann et al. 2022). The Eixample neighborhood in Barcelona, illustrates how a well-connected grid can enhance mobility and urban vibrancy (Nello-Deakin et al. 2024). Nevertheless, reducing urban design to limited connectivity metrics may overlook essential aspects including aesthetics, historical significance, and temporal transformations (Carmona et al., 2010).

Graph measures, such as redundancy and robustness, that are employed in resilience research, particularly in model disruptions, such as natural disasters and network failures (Zeng et al., 2022). This analytical framework, therefore, is essential for identifying systemic vulnerabilities (Jenelius and Cats, 2015). Urban computing related to graph theory involves the integration of big data, GPS, and social media information to construct real-time graph-based models of human movement of urban networks (Zheng, 2019). Despite their effectiveness, these approaches raise concerns regarding privacy, algorithmic bias, and the marginalization of digitally invisible individuals (Kitchin, 2014).

Each application of mathematical, computational, or morphological displays a risk of reductionism, frequently neglecting human agency, cultural context, and systemic injustice. The challenge lies not in eliminating the abstract model, but in situating it within the appropriate context.

## 2. METHOD

### Case Study: Downtown Manhattan, New York City CBD

The Lower Manhattan zone is located south of Murray Street and the Brooklyn Bridge and includes the southern side of Chinatown and the Lower East Side. It constitutes the financial district or the principal CBD of the United States (U.S. Federal Transit., 2004). In 2000, Lower Manhattan's street urban design reflected was colonial and lined with low-rise buildings alongside mid-to late-nineteenth-century structures, Art Deco towers, and tall, modern skyscrapers. Residential buildings are distributed within mixed-use office buildings and at the ninety-two-acre Battery Park City. The institutional uses include the American Indian National Museum, Federal Hall, and St. Paul's chapel. The southern streets near the seaport contain a historic market which includes retail shops and restaurants, a museum, and river piers for pedestrians. The narrow-irregular street layout caused traffic congestion and heavy pedestrian flow (U.S. Federal Transit 2004; Willis 2023). Figure 1. Selected study area.

### Conceptual framework and method selection rationale

The selection of analytical methods in this study is guided by a conceptual framework that connects selected urban dimensions to mathematical abstractions. This framework is operationalized through methods that address the core components of the CBD's complexity (see table 2).

This multi-method pipeline is not merely a collection of tools but an integrated system in which each component addresses a different aspect of urban phenomena: structure, economics, morphology, dynamics, and behavior. Figure 2 outlines the relationships among mathematical coding, graph theory, and urban structure indices that form the proposed analytical model. The model comprises a set of measurement equations, as illustrated in Table 3.

The analysis of urban structure using space syntax identifies optimal locations for specific activities. Economic and game theory models explain why and how these activities occur, while vulnerability and complexity analyses reveal the constraints and resilience of the CBD's urban structure. This multi-method approach enables triangulation, allowing the results

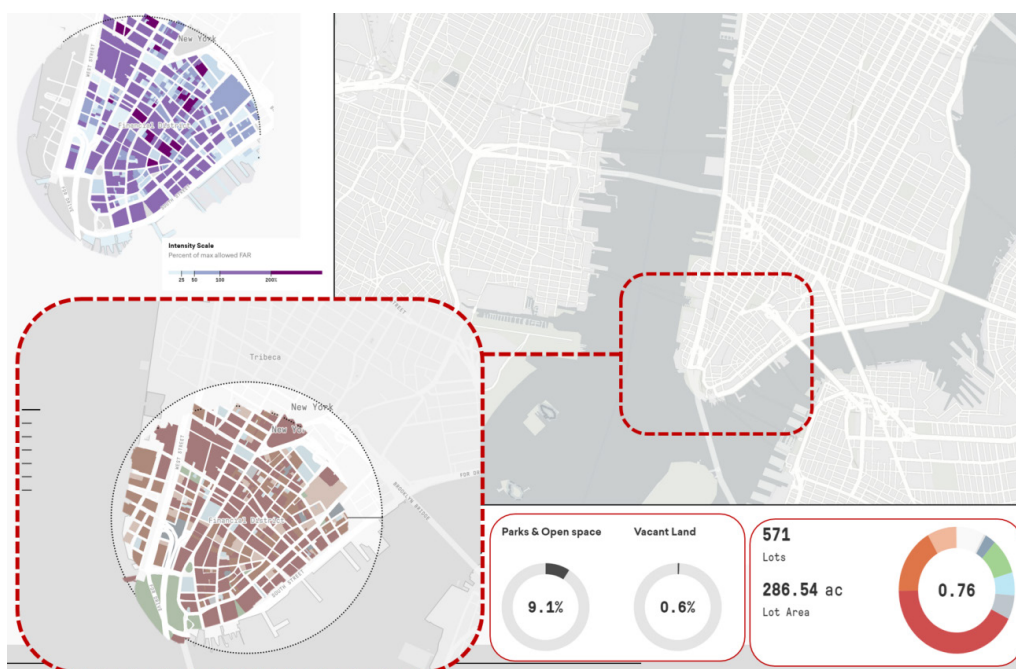


Figure 1. Lower Manhattan CBD (New York City downtown). Source: Morphocode Explorer.

of one analysis, such as the high integration of eastern cores, to be cross-checked against another, for example, economic activity patterns and pedestrian flows.

#### Data sources, quality control, and limitations

Data for this study were derived from several sources, including official databases (NYC Open Data and NYC DOT), real estate market reports, and urban platforms (Rocket Homes, Morphocode, First Street). These sources present several potential challenges:

1. Heterogeneity and resolution: Data were collected at walking distances levels (within 800 meters).
2. Inherent biases: Real estate and retail data can reflect market biases and may underrepresent informal economic activities. Pedestrian counts are limited to specific bi-annual surveys and may not capture daily or seasonal fluctuations.
3. Uncertainty in risk data: Climate risk probabilities are model-based estimates subject to their own assumptions and uncertainties. Our vulnerability score should therefore be interpreted as a relative indicator of risk, not an absolute forecast.

We have taken care to use the most recent and authoritative sources available, but the findings should be interpreted with an understanding of the inherent limitations and potential errors associated with these

secondary datasets.

#### Measurement

A study-area graph was constructed, consisting of nodes and paths with hierarchical prefix-based labeling to define the general system of Lower Manhattan CBD. The urban system vulnerability to probable risks is measured. Since the economic factor of the CBD is vital according to urban structure theories, the measurement of vulnerability focuses on the risk of climate factors like flooding, heat, wind, and fire. The recent data needed for the analysis is available for Lower Manhattan. Lower Manhattan's probability of occurrence (P) values are (30%) for flooding, (12%) for fire, (98%) for wind, and (51%) for heat in thirty years (www.redfin.com). System State (S) values of land-uses are for residential use (75%), for commercial (127.4%), for social (0.021%), and for roads (20%). Total system state (S) values are calculated using the summation of the land-use values which equals to (222.421). The system response capability (R) value equals to (23.2%) while trigger threshold (T) represents the percentage of properties at risk that its value equals to (37%). Vulnerability adjustment coefficient (C) equals to (-1) so the properties at risk are  $(1 - 0.37 = 0.63 \text{ or } 63\%)$  (Zhang et al., 2016; Alam et al., 2012; Graham, Debucquoy et al., 2016). The values of the coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  equal to (1) as a constant (Morphocode Explorer; Downtown Manhattan, NY Flood Map). The risk of inclination (R)

**Table 2. Connection between Urban Dimensions and Mathematical Abstractions**

Indices	Definition	Purpose
Graph Theory & Space Syntax	Graph theory provides the abstract language to represent the urban network, while space syntax translates this into measurable configurational properties (integration, connectivity, depth).	They were selected to answer the primary research question: "What is the inherent spatial logic of the CBD?" They directly link the physical grid to predicted movement and economic function.
Vulnerability Assessment (General System Definition)	This was employed to quantify the systemic fragility of the CBD.	By modeling the urban system as a set of interacting components (land use, hazards, response capacity), it moves beyond descriptive risk mapping to a probabilistic assessment of failure, addressing the study's aim to elucidate "susceptibility to potential threats."
Zipf's Law	This principle was applied not as a linguistic curiosity, but as a tool to measure economic gravity and hierarchy.	In an urban context, it models the influence of a central economic core (the Financial District) on the adjacent zones. It is used for quantifying the economic interaction between tracts i and j.
Markov Chains	This method was selected to model dynamic urban economic processes.	The value of urban land, housing supply, and retail establishment are turned into states of transition using Markov chains to define probabilistic state changes over time.
Q-Analysis & Fractal Geometry	These methods were chosen to capture the morphological complexity and scaling of the urban form.	Q-analysis reveals the higher-dimensional relationships between buildings and streets that simpler graphs miss. Fractal geometry quantifies the self-similar, organically evolved nature of the urban fabric. They were integrated to answer how the "interconnectedness of shape, function, and temporal change" manifests across different scales.
Game Theory (Nash Equilibrium)	This was not used to model rational actors in a classic economic sense, but to analyze the emergent equilibrium in pedestrian flow	On a crowded street network, individual route choices lead to collective congestion patterns. Nash Equilibrium helps identify stable states where no pedestrian can unilaterally improve their commute, thus revealing temporally variable bottlenecks and optimal flow distributions.
Heuristic Pathfinding Algorithms (A*, Dijkstra's, etc.)	These were implemented to diagnose the navigational performance and latent structure of the street network.	By testing different search strategies, we can infer the network's inherent complexity, its semi-lattice properties, and identify critical paths that influence overall connectivity.



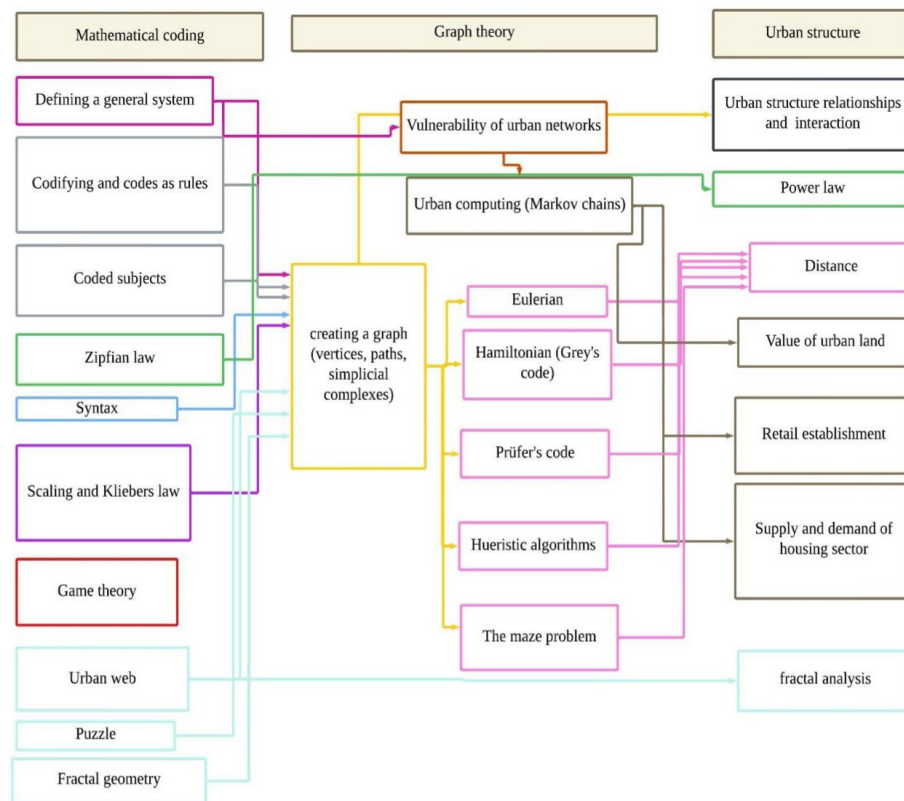


Figure 2. The proposed analytical model illustrates the relationships among mathematical coding, graph theory, and urban structure.

(the slope of risk) is estimated as (23.2%) in Manhattan downtown (Downtown Manhattan, NY Flood Map and Climate Risk Report | First Street).

Codification, using rules, is measured using Prüfer's code. It is applied to tree graphs to investigate the isomorphism within urban structure. The Manhattan graph contains a lattice structure with leaf nodes labeled (L301- L320), where "L" represents a leaf node, (300) is the total number of vertices, and 1 to 20 correspond to the individual leaf nodes. The first step is to convert the lattice-leaf graph to a spanning tree using Kruskal's algorithm from the root node to the destination node (Galbrun, E. Pelechris, K. and Terzi, E. 2016). Zipfian law is applied to Lower Manhattan's CBD. It is used to measure the impact of the seaport financial district on the adjacent zone (Zipf, 2016). The first step is to define two tracts (i,j) to apply the Zipfian equations for generalized harmonic series and the relationship between population and distance equations:  $\{(3.1), (3.2), (3.3)\}$ .

Tract (i) : the tract total area is (295.12) acres, the vacant land is (1.8), basic land-uses are (68.9), retail is (178), and household is (24.4). population (P) within tract (i) is nearly (36.442) with density of (73.3 people per acre).

The employment (E) ratio is (92.77%).

Tract (j): the tract total area is (321.33) acres, the vacant land is (4.4), basic land-uses is (81.7), retail is (198.6), household is (35.8). population (P) within tract (j) is nearly (67.783) with density of (136.4 people per acre).

A higher  $S_n$  value ( $>2$ ) indicates more evenly distributed contributions of competing tracts, while a lower  $S_n$  value ( $\leq 1$ ) suggests more control by the leading tracts (Bettencourt et al. 2007).

Primal, dual, polygon-centroid, and Q-analysis measurement were applied to Lower Manhattan, following Hillier's theory of space syntax (Hillier and Hanson 1984). First, the primal problem was tested through traditional analysis of space syntax representing the relationship between the streets (paths) and their related junctions (nodes). Second, the dual problem analyzed the relationship among these junctions (nodes) through their streets (paths) as a morphological representation. Locations and linear features (nodes and paths) are the key concepts in morphological relations. The analysis contains the dual syntax graph (300 nodes /498 paths), the primal-path syntax graph (496 nodes/503 paths), and the primal-polygon

Table 3. Equations used for measurement.			
Indices	Equation	Notes	Reference
General system definition	$V_{score} = \alpha \cdot \ln(1 + P_{failure} \cdot S_{state}) + \beta \cdot \tan(R) + \gamma \cdot \frac{1}{1 + \exp(-T)} - \delta \cdot (1 - C) \dots (1)$ $P_{failure} = \sum_{k=1}^n \omega_{Po} \cdot Po \dots (2)$ $S_{state} = \sum_{k=1}^n \omega_{So} \cdot So \dots (3)$ $R_s = \sum_{k=1}^n \omega_{Ro} \cdot R_o \dots (4)$ $T_{threshold} = \sum_{k=1}^n \omega_{To} \cdot To \dots (5)$ $C_v = \sum_{k=1}^n \omega_{Ck} \cdot Co \dots (6)$	The vulnerability is calculated using sub-equations	(Sun et al., 2024)
Codifying and codes as rules	Prüfer's code	Calculated using heuristic algorithms	(Prüfer, 1918)
Zipfian law (least effort principle)	$p \times S_n = P/1^p + P/2^p + P/3^p + \dots + P/n^p \dots (7)$ $\text{density}_D = P^p / D^2 \dots (8)$ $\gamma/C = P_1 \times P_2 / \text{Density}_D \dots (9)$	Zipf's lemma expressed economic problems of raw materials and then converting them to consumable goods. As a result, unorganized people turn out to be more organized.	(Zipf, 1949)
Syntax	Connectivity (i) = $\sum A_{ij} \dots (10)$ Angular connectivity (i) = $\sum A_{ij} \times \cos(\theta) \dots (11)$ Integration (i) = $N - 1 / (\sum D_{ij}) \dots (12)$ Weighted integration (i) = $1 / (\sum D_{ij} \times l_{ij}) \dots (13)$ Total depth (i) = $\sum D_{ij} \dots (14)$	Hillier's primal graph Batty's dual graph Polygonal centroid Atkin's Q-analysis	(Hillier & Hanson, 1984)
Markov chain	$S(t+1) = P_{s1s2} \times S(t) \dots (15)$	Markov formula of transition from one state to another	(Badshah et al., 2024, p.5)
Scaling law and Klieber's law	$Y(t) = Y_0(t)N(t)^\beta \dots (16)$ $B = \beta M^{3/4} \dots (17)$		(Kleiber, 1947; West et al., 1997)
Coded subjects	$d[v] = \min(d[v], d[u] + w(u, v)) \text{ for all edges } (u, v) \dots (18)$ $f(n) = g(n) + h(n)$ $g(n): \text{cost from start to node } n,$ $h(n): \text{heuristic estimate to goal} \dots (19)$	Dijkstra algorithm A* algorithm Greedy algorithm Bidirectional search algorithm	(Dijkstra, 1959; Hart et al., 1968)
Game theory and ABM	Payoff matrix	Nash equilibrium	(Nash, 1950)
Puzzle	Heuristic algorithms		(Pearl, 1984)
Urban web (the fractal dimension)	$D = \lim_{\epsilon \rightarrow 0} \log N(\epsilon) / \log(1/\epsilon) \dots (20)$ $C(r) = \lim_{N \rightarrow \infty} 1/N^2 \sum_{i=1}^N \sum_{j=1}^N H(r -  x_i - x_j ) \dots (21)$ $C(r) \propto r^{D_2} \dots (22)$	Counting box and correlation	(Grassberger & Procaccia, 1983)

centroid syntax graph (127 nodes/345 paths). Because the primal space syntax analysis using DepthmapX software does not consider the relationships among land-uses and buildings heights, the dual syntax graph is used to represent the visual relationships among spaces. The primal-path syntax graph represents spatial interactions among these places as prediction of movement flow. Finally, the primal-polygon centroid syntax graph predicts interactions among buildings.

The incidence matrix was built for Q-analysis (Atkin 1974). The resulting Q-connectivity (the sequence of simplices) was analyzed using several Q-algorithms. The incidence matrix displays the relationship between building heights and the corresponding street segments, examining the three-dimensional representation of building heights alongside the isovist or the visual field of a certain point in urban space (Hillier 1984), which measures visibility in two dimensions. The incidence matrix is a binary relationship: a value of 1 indicates a relationship, and 0 indicates no relationship, covering 184 buildings and 508 paths. The symbol  $\lambda$  represents this relationship. Using Gephi 0.10.1 software, Q-analysis was conducted to interpret the resulting values and visualize the simplices. The rows represent the relationship between each building height and its adjacent paths as n-dimensional simplices, while the columns represent the total number of paths connected to each building, expressed as the summation of q-connected values.

Markov chains are used to predict urban land values, the supply and demand in the housing sector, and retail establishments within the CBD (Isaac & O'Leary, 2011). Land value data, expressed as sale price per square foot, provides an indication of market health and trends in property values. The housing supply decreased from 5,964 to 5,106 units between November and December 2024, which could drive prices upward if demand remains stable or increases. The average sale time increased from 82 days in December 2023 to 133 days in December 2024, indicating slower sales (Manhattan, New York Housing Market Report, December 2024 – Rocket Homes). The prediction of future urban land values is based on the following transition matrix, where  $V_0 = [0, 0, 1, 0, 0]$ , and the transition states are:  $T_V =$

$$V(t+1) = [0.0, 1.0, 0.0] \times T_V \dots \dots \dots (23)$$

$$\begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 & 0.0 \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.0 \\ 0.1 & 0.2 & 0.5 & 0.1 & 0.1 \\ 0.0 & 0.1 & 0.3 & 0.4 & 0.2 \\ 0.0 & 0.0 & 0.1 & 0.3 & 0.6 \end{bmatrix}$$

The prediction of housing supply and demand using

a Markov chain is based on historical supply data and winter supply drop percentages between 2015 and 2025. The number of housing units in December 2015 was 4,000, in December 2023 it was 11,000, and in December 2024 it was 8,000. Winter supply declines were recorded at -14% in 2015, -15% in 2023, and -17% in 2024 (Lower Supply, Higher Stakes: Manhattan's Largest Inventory Drop in Over a Decade). The prediction of future housing supply and demand values is based on the following transition matrix, starting from the stable market state, where  $H_0 = [0, 0, 0, 0, 1]$ , with the transition states as follows:

$$T_H = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.5 & 0.5 \\ 0.0 & 0.0 & 0.5 & 0.5 & 0.0 \\ 0.2 & 0.2 & 0.4 & 0.2 & 0.0 \\ 0.0 & 0.2 & 0.2 & 0.4 & 0.2 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.5 \end{bmatrix}$$

$$(t+1) = [0.0, 0.0, 0.1] \times T_H \dots \dots \dots (24)$$

The prediction of retail establishments in Lower Manhattan is based on the current distribution in 2024: food, drinks, and entertainment (21%); retail (29%); services (32%); and community facilities (18%). Storefront turnover by occupant type between 2020 and 2024 shows that food, drinks, and entertainment experienced 1,350 more openings than closures; retail had 540 more closures than openings; community facilities had 70 more closures than openings; and services had 2,720 more closures than openings (Storefront Activity in NYC Neighborhoods, 2024). The prediction of future retail establishments is based on the following transition matrix, where  $S_0 = [0.21, 0.29, 0.32, 0.18]$ , with the transition states as follows:

$$T_R = \begin{bmatrix} 0.5 & 0.3 & 0.15 & 0.05 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.05 & 0.15 & 0.2 & 0.6 \end{bmatrix}$$

$$S(t+1) = [0.21, 0.29, 0.32, 0.18] \times T_R \dots \dots \dots (25)$$

These indices are applied to Lower Manhattan due to the availability of recent data. The combination of the scaling law with Kleiber's law produces an equation that incorporates three aspects vital to the CBD: economic output, economic input factors, and underlying biological principles (Bettencourt et al. 2007; Kleiber 1947). The resulting model is as follows:

$$Y(t) = Y_0(t) [\beta M^{3/4}(t)]^\beta \alpha \dots \dots \dots (26)$$

Where:

$Y(t)$ : Economic output at time  $t$

$Y_0(t)$ : Initial economic output at time  $t$

$B$ : Kleiber's law coefficient

$M(t)$ : Relevant input factor (e.g., population, labor, capital) at time  $t$

$\alpha$ : Exponent that determines the sensitivity of the economic output to the input factor.

According to New York City (NYC) data from 2023-2025, the values for non-farm employment (total) are 1.0, 0.9, 0.4; wages (total) are 5.3, 4.9, 4.1; personal income is 4.7, 4.3, 3.8; unemployment rate (percent) is 4.1, 4.2, 4.3, and the composite index are (4.0, 2.9, 2.4). The weights for non-farm employment or wages ( $E$ ), personal income ( $PI$ ), unemployment rate ( $U$ ), and composite inflation effect ( $CPI$ ) weights are (35%, 25%, 20%, 10%, 10%) respectively.

$Y_0(2022) = 2$  trillion representing NYC's Gross Metropolitan Product, beta value  $\beta=0.75$ , alpha  $\alpha=1.2$  represents the typical elasticity of economic output to labor in macroeconomic models (Dadayan 2024). Coded subjects in Lower Manhattan are measured. The A\* (star) algorithm is used to solve the maze problem and extract the Hamiltonian (Gray's Code) for pathfinding (Hart, Nilsson, and Raphael 1968). Dijkstra's algorithm finds the Eulerian path in a weighted graph using a greedy approach (Dijkstra 1959). The greedy algorithm is used for the Prüfer code extraction because of its ability to construct the spanning trees and isomorphism checking to compare structures. Coded subjects are measured between the starting node ( $A$ ) and destination node ( $B$ ) in Lower Manhattan where the shortest path is selected among other paths in the urban network.

Game theory, or the interaction among pedestrians, is measured using Nash equilibrium in Lower Manhattan (Nash, 1950). The study area encompasses the following streets: Broad Street, Broadway, Fulton Street, Lexington Avenue, Trinity Place, and Wall Street. According to NYC Department of Transportation, the highs and lows in 2024 of pedestrian density according to months ranges between 79.099 on September 7 and 19.328 on May 7. Trinity Place recorded the lowest pedestrian density of 217.217 while Wall Street recorded the highest density value of 433.016 (Bi-Annual Pedestrian Counts | NYC Open Data). Nash equilibrium represents the pedestrian traffic patterns where no single street/time segment can unilaterally improve its pedestrian flow. Pedestrian traffic is measured in three steps: 1. Calculate the mean pedestrian count for each (street/time). 2. Identify the strategic interaction points. 3. Determine which

street at certain time can optimize the pedestrian flow. Three steps of the calculation methodology are used to measure the pedestrian interaction: the payoff of pedestrians or the mean value of pedestrian count, and Nash equilibrium. Nash equilibrium includes: the higher payoff values than average and low variability in pedestrian counts, and the strategic stability score composed of metric considering payoff.

Puzzle examination of the urban network results includes the maze problem, space syntax, and intersection-type metrics for Lower Manhattan CBD. The maze problem uses heuristic algorithms between any two nodes for pathfinding. It is also related to the intersection types that facilitate or obstruct navigation through the urban environment. The space syntax analysis of the primal graph is compared to the four morphological types of urban structure suggested by Hillier (Hillier 1996) to recognize the genotypes. The urban web (fractal dimension) is calculated using (Fractalyse3-0.9.1) software for Lower Manhattan CBD (Frankhauser 1994). Three dimensions were analyzed: box counting, radial of all points, correlation, and dilation). The raster input of the image was imported in TIFF format from Google Earth Pro at high resolution. The measurement software and parameters are detailed in Table 4.

### 3. RESULTS AND DISCUSSION

The results for Lower Manhattan CBD encompass a multi-faceted analysis, including vulnerability assessment, spatial syntax, economic modeling, and pedestrian movement patterns.

General system definition (vulnerability to probably risk)

The graph-based system definition enabled a quantitative vulnerability assessment. The overall vulnerability score for Lower Manhattan was calculated at 2.13, indicating a very high-risk level. The main contributing factors include a higher-than-average probability of hazard occurrence ( $P=0.47\%$ ), a diverse land-use system state ( $S=55.60$ ) that influences vulnerability, and a moderate system response capability ( $R=23.2\%$ ). Critically, the trigger threshold ( $T=37\%$ ) reveals that over a third of properties are at risk from storm surge, and the vulnerability adjustment factor ( $C=0.63$ ) confirms that the urban system's characteristics exacerbate overall risk. This synthesis confirms Lower Manhattan's high susceptibility to climate-related disruptions (see Figure 3).



**Table 4. Software and parameters employed for urban network measurement (freely available).**

Indices	Processing	Analysis
Space Syntax	Axial and segment maps were constructed in dxf. format.	Space syntax analysis is run using UCL Depthmap X software
Q-Analysis	The incidence matrix was constructed in CSV format. The matrix is a binary structure where a value of 1 was assigned if a building facade was directly adjacent to a street segment, operationalizing visual accessibility.	Gephi 0.10.1 software is used to visualize the matrix relationships
Fractal analysis	Google earth map in TIFF format of resolution 300 dpi	Fractal dimensions were calculated using Fractalyse 3.0.9.1. The box-counting method
Pathfinding algorithms	The heuristic function was applied $h(n)$ was set as the Euclidean distance to the goal, appropriate for a relatively planar urban grid.	Pathfinding visualizer (online platform) by Github.



a



b



c



d



Figure 3. General system definition: (a) and (b) primal syntax graph of Lower Manhattan CBD with hierarchical prefix-based labeling, (c) dual syntax graph of Lower Manhattan CBD, and (d) tessellation of Lower Manhattan CBD. Source: Authors.

#### Zipfian Law CBD Interaction

Zipfian law analysis quantified the influence of the financial district on an adjacent zone (Tracts i and j). The high value of the generalized harmonic series ( $S_n=53,387$ ) indicates a significant, yet adjusted, contribution from both tracts to the overall economic influence, with Tract i carrying more weight (Zipf, 2016; Bettencourt et al., 2007). The economic interaction measure ( $y/C$ ) implies dominate gravitational influence from the center of the CBD to the neighboring region, thus emphasizing its central business role.

#### Syntax analysis

Space syntax analysis defines and empowers the understanding of spatial configuration and accessibility in urban scales (Berhie, G. K., & Haq, S. 2017; Alobaydi et al. 2025; Al Ani M. 2014).

#### Primal syntax graph

Results from the analysis of path connectivity and integration revealed hierarchy in connectivity and integration. Path connectivity values varied from minimum value one for dead-end paths to maximum value seven as shown in FiDi/Financial District. Higher values of angular connectivity measures (maximum of 6.04) showed efficient paths for flow despite low angular measures. Topological integration analysis showed accessibility to paths on one side of Lower Manhattan at 500 m radius than to paths on any side of Lower Manhattan at 800 m radius. This shows a complex network for flow paths as depicted in Figure 4.

#### Dual path syntax graph

This analysis represented streets as nodes and intersections as links, emphasizing the role of position within the network. Connectivity scores were higher at eleven for major points of congregation and areas of openness linked to commercial and public land use. Lower scores of two were linked to residential areas instead. The results confirmed that highly accessible locations, identified by interlinking junctions, are concentrated at the center of Lower Manhattan, which hosts key public and institutional functions, as shown in Figure 5 below.

#### Primal-polygon centroid syntax graph

This analysis was conducted to establish relationships

between urban blocks. There was high connectivity between the center and peripheral areas, particularly on the western side. In contrast, areas of high integration values, which denote high accessibility, were situated on the eastern side, which contains business and mixed development areas. This suggests a distinction between highly connected blocks and highly accessible blocks, as depicted in figure 6 below

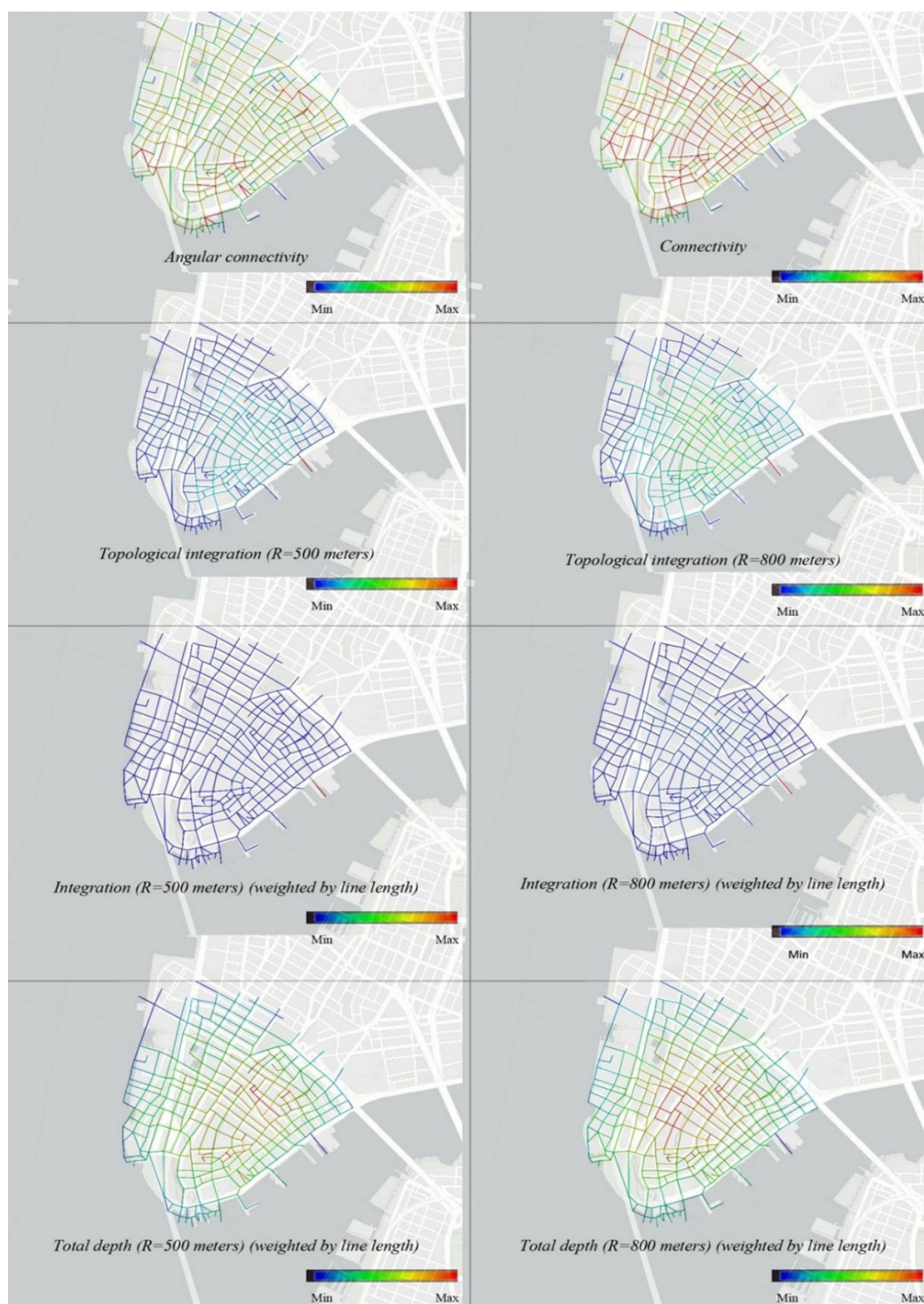


Figure 4. Results of the primal syntax graph of Lower Manhattan (CBD). Source: authors' calculations using UCL DepthmapX software.

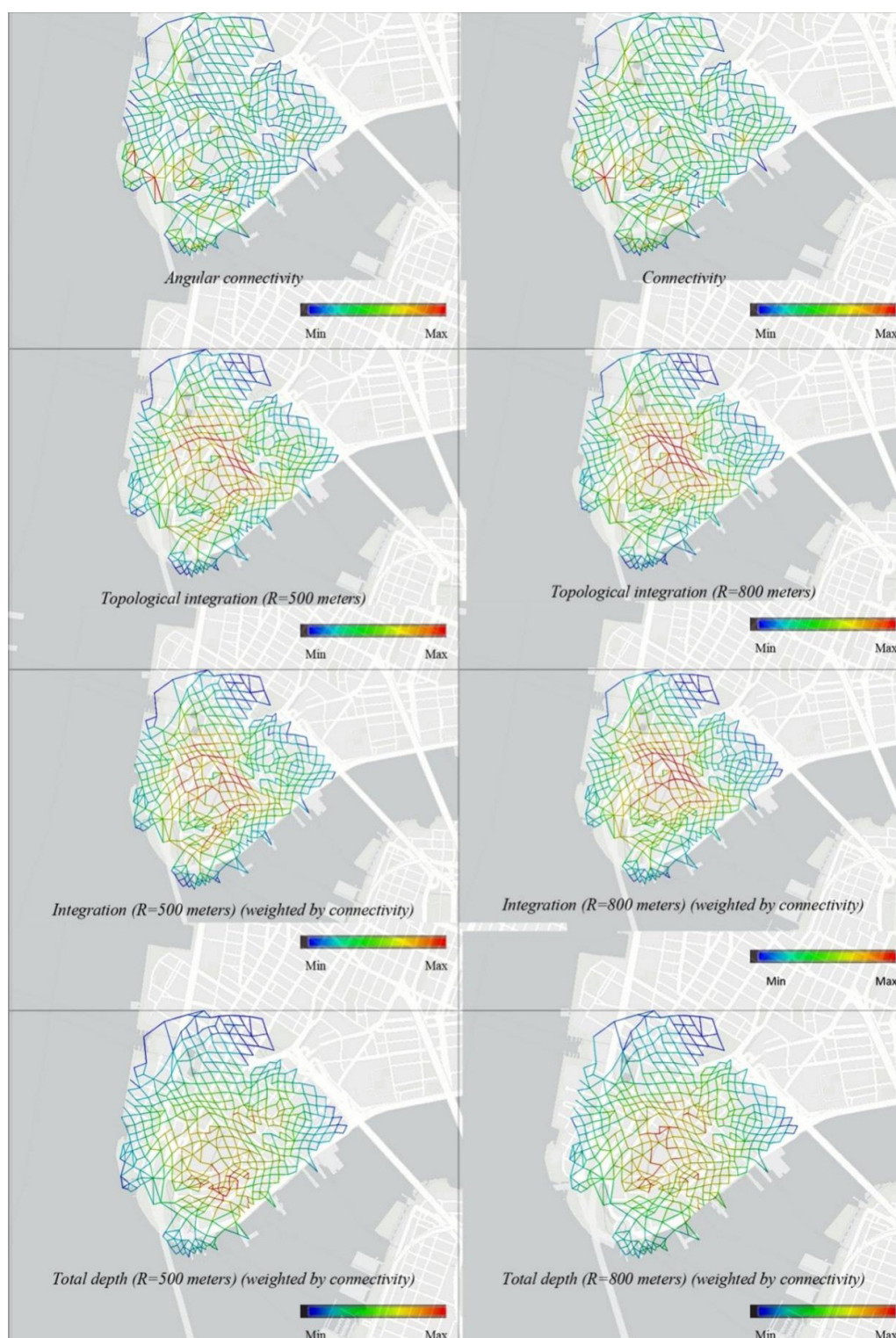


Figure 5. Analysis results of the dual-path syntax graph of Lower Manhattan (CBD). Source: authors' calculations using UCL DepthmapX software.



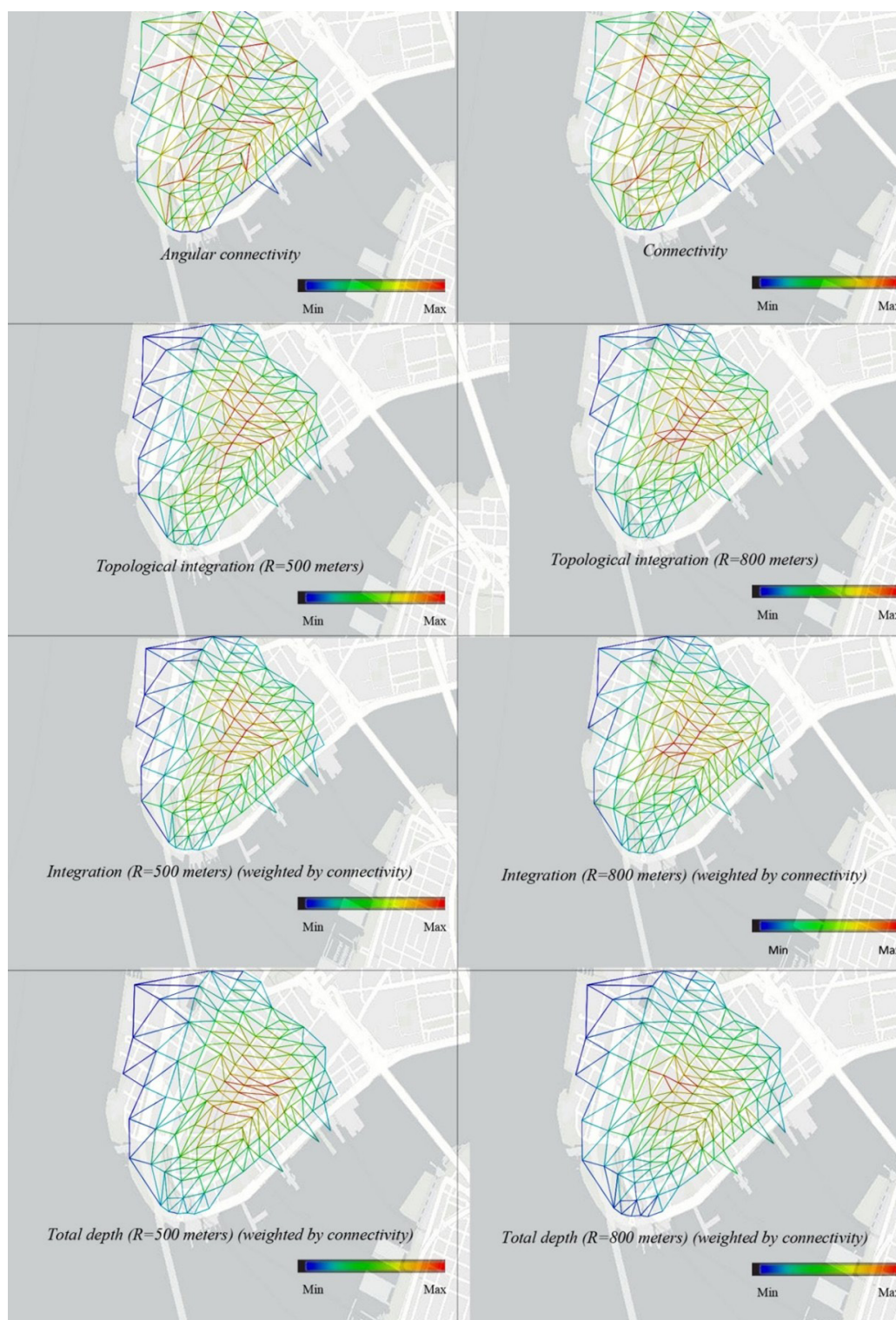


Figure 6. Analysis results of the primal-polygon centroid syntax graph of Lower Manhattan (CBD). Source: the authors using UCL DepthmapX software.





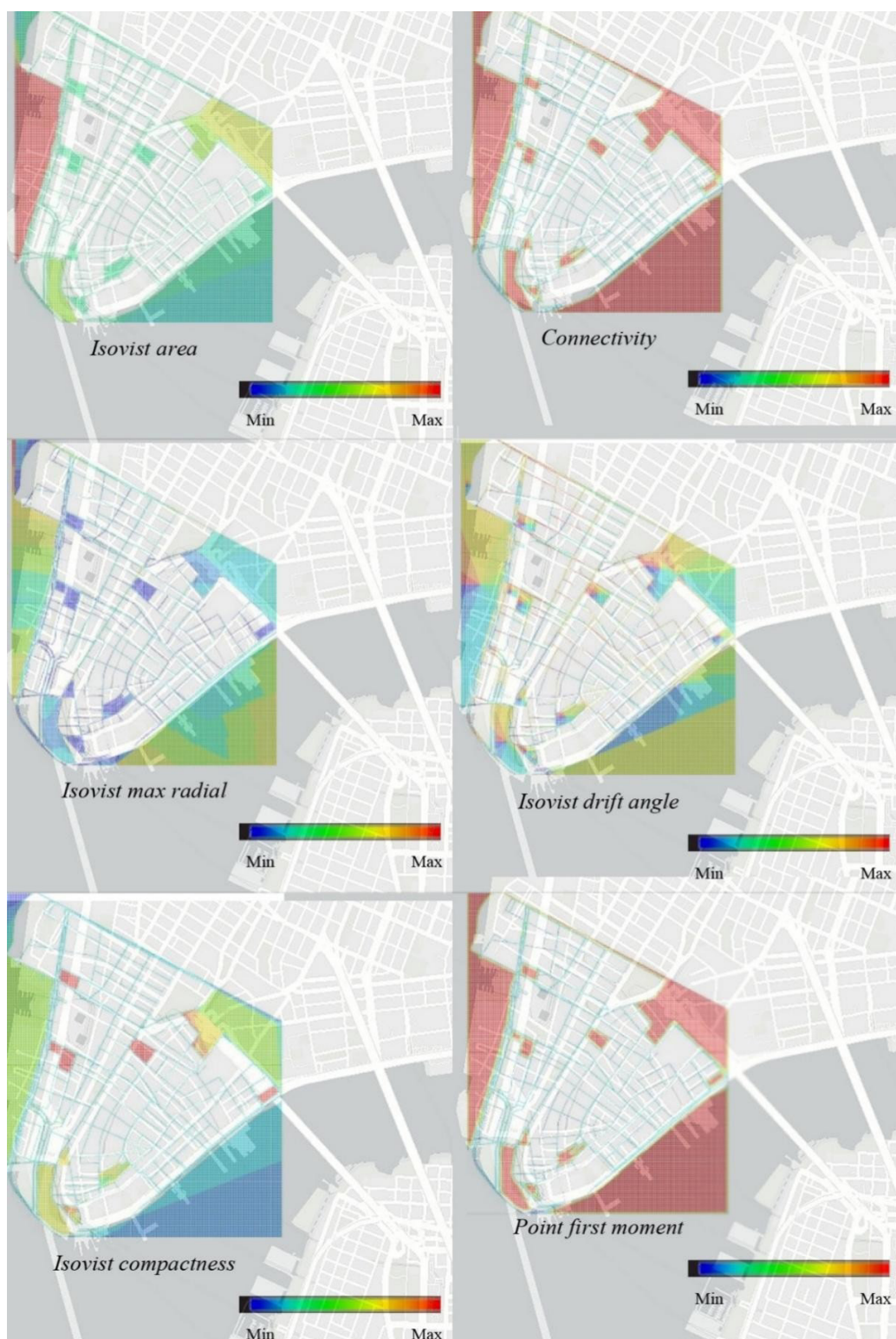


Figure 8. Analysis results of visibility graph analysis (VGA) of Lower Manhattan (CBD). Source: the authors using UCL DepthmapX software.

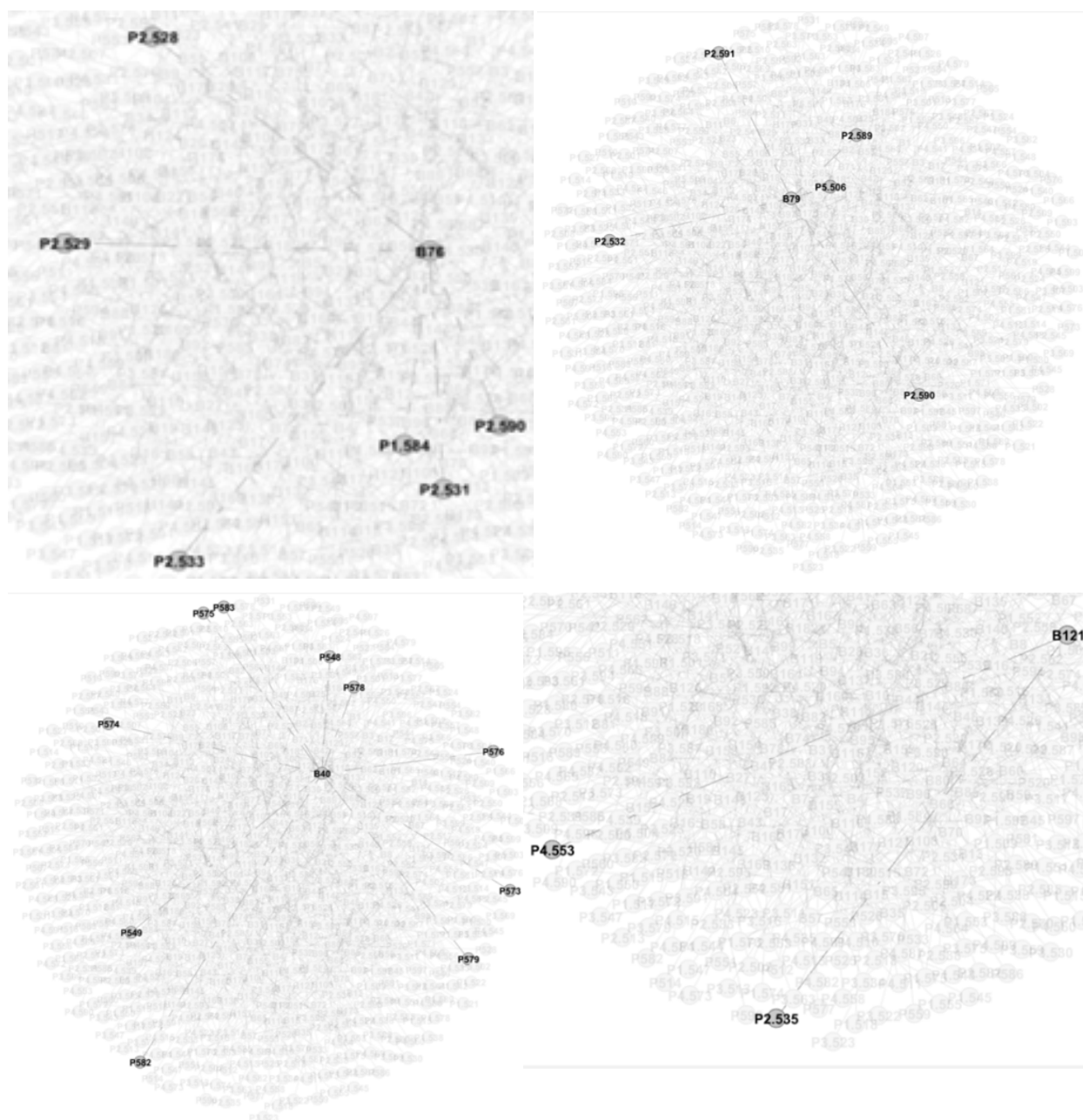


Figure 9. Results of Q-analysis visualization using (Gephi 0.10.1) software. Source: the authors.

#### Markov chain (urban economics)

Markov chain modeling forecasted future trends in urban land value, housing, and retail as illustrated in Figure 10.

Urban Land Value: Predictions indicate a market consolidation around the mid-range value of \$1.2 million, which is expected to comprise approximately

36.4% of the market in five years. This measurement indicates stabilization and an increase in values.

Housing Supply/Demand: The model predicts a supply focused on the mid-range (around 6,000 units), which indicates a move toward higher-density housing projects. Demand is expected to remain steady.

Retail Establishment: A significant shift in the retail

landscape and a move towards service-oriented (30.6%) and experiential (24.7% for food/drink/entertainment) establishments. The stable share of community facilities (15.4%) points to a sustained need for local amenities amidst this evolution.

#### Coded subjects (pathfinding)

The application of heuristic algorithms (A\*, Dijkstra's, Greedy, Bidirectional) for pathfinding confirmed the semi-lattice structure of the urban network. Analysis of the minimum spanning tree derived from the incidence matrix (639 edges) revealed twenty-one leaf nodes and



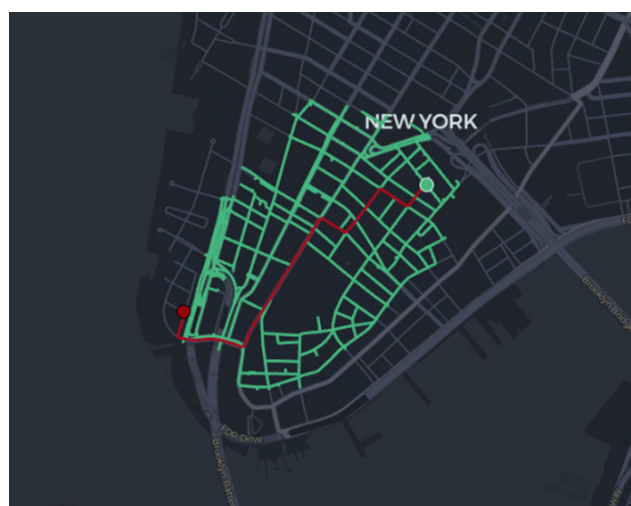
Figure 10. Results of Markov chain probabilities after (5 years). Source: the authors.

#### Scaling and Kleiber's law (economic output)

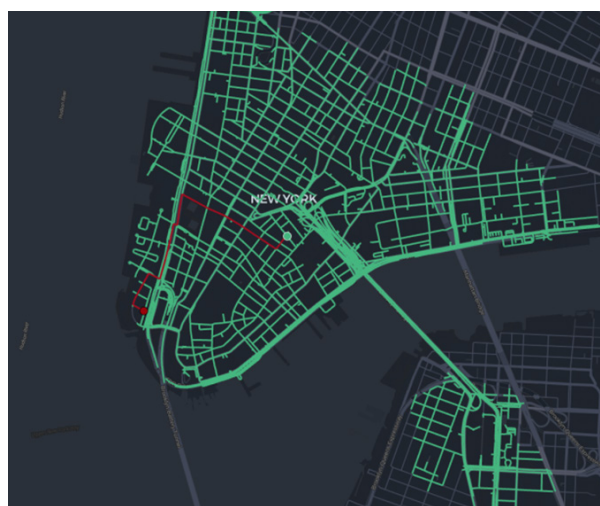
Applying a combined scaling-Kleiber's law model projected New York City's Gross Metropolitan Product (GMP) to decline from \$2.20 trillion in 2024 to \$1.80 trillion in 2025 (Bettencourt et al. 2007; Kleiber 1947). This projected economic contraction is driven by forecasted slowdowns in key input factors: employment growth (0.4%), wage growth (4.1%), and personal income growth (3.8%), combined with a rising unemployment rate (4.3%).

numerous isomorphic sub-trees (thirty-two of degree three, six, of degree four), indicating a high degree of structural repetition and similarity within the urban fabric, which influences navigation and connectivity as shown in Figure 11.

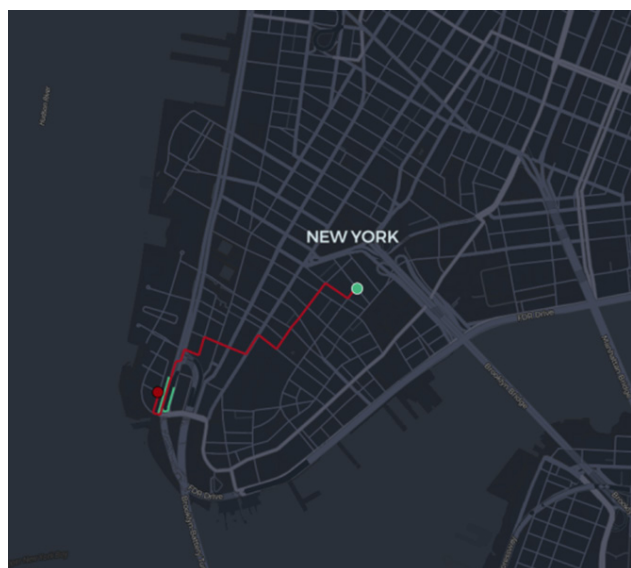




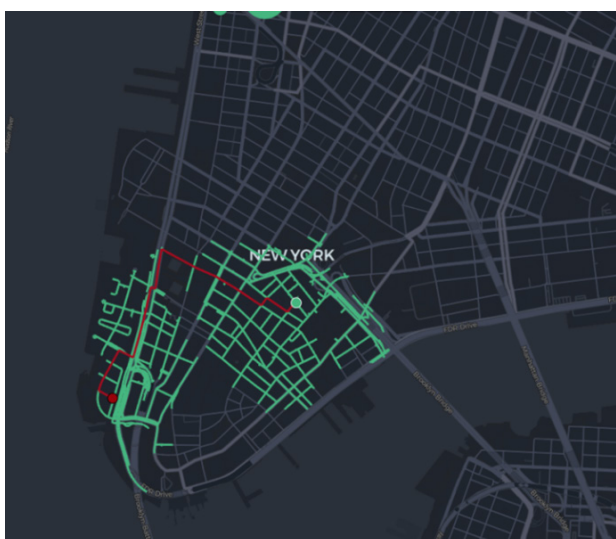
A\* algorithm



Dijkstra's algorithm



Greedy algorithm



Bidirectional algorithm

Figure 11. Results of the heuristic algorithms of Lower Manhattan. The chosen path is located between the starting node and the destination node, where the green path is the selected among other paths in the urban network. Source: the authors using Map Pathfinding Visualizer.

#### Game theory (pedestrian payoffs)

Nash equilibrium analysis of pedestrian flow data revealed distinct congestion patterns across key streets and times (Nash 1950). Fulton Street consistently showed the highest payoff (1.00), indicating the least congestion across all time periods. In contrast, Broadway exhibited significant congestion, particularly in the afternoon. Wall Street's higher payoff occurs in the morning periods compared to afternoons, and illustrates a clear temporal shift in pedestrian congestion, valuable for urban mobility planning as shown in Figure 12.

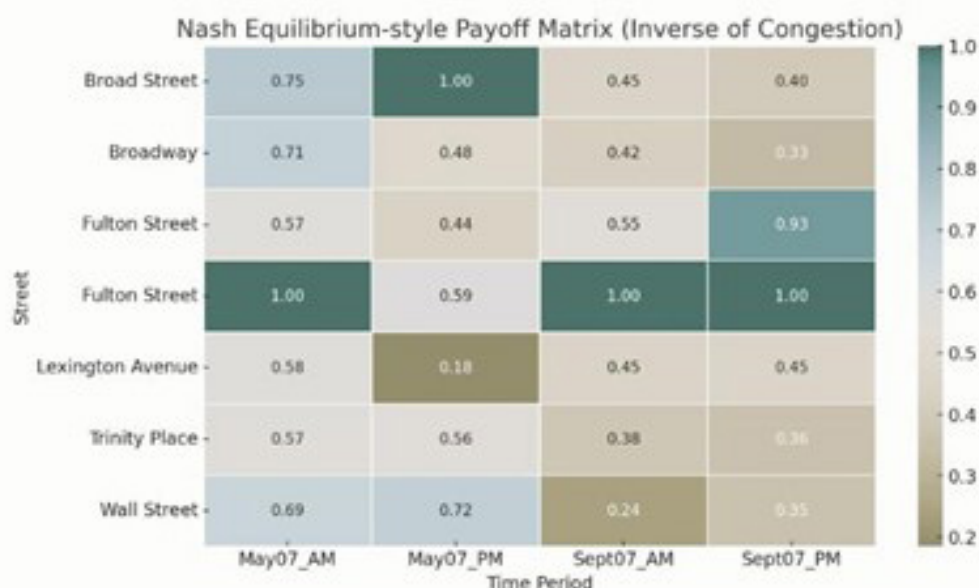
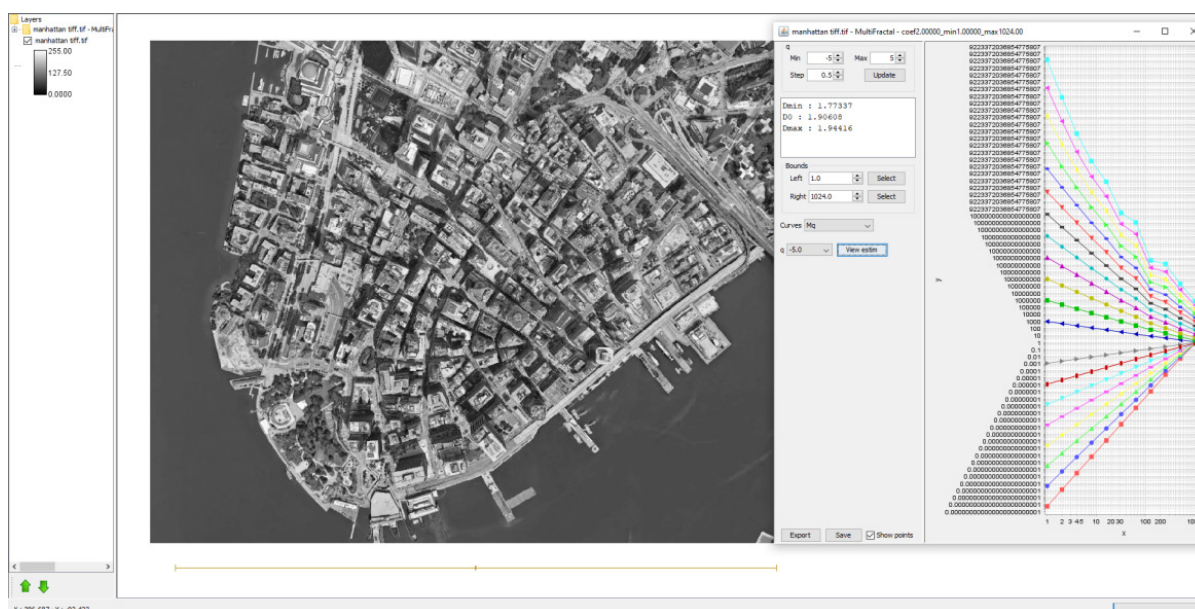


Figure 12. Results of the Nash equilibrium pay-off matrix. Source: the authors.

### Puzzle (maze problem and genotype)

Analysis of the maze problem and space syntax revealed the urban genotype of Lower Manhattan to be a "regular non-geometric heart" structure. The highly accessible "shallow" structure was in the eastern commercial and institutional core, while the less accessible "deep" structure was found on the western side, correlating with mixed-use and residential areas. This defines a clear spatial hierarchy of movement and function.

Urban web (complexity)Fractal dimension analysis confirmed the extreme spatial complexity of Lower Manhattan's CBD. The box counting method yielded a very high dimension ( $\sim 11.07$ ), reflecting intricate layering of the built environment. The radial analysis dimension ( $\sim 1.96$ ) and dilation dimension ( $\sim 1.84$ ) both pointed to a dense, space-filling, and highly interconnected urban core. The high R-squared values and statistical significance of these results robustly confirm a multifractal, hierarchical, and organically evolved urban form as shown in Figure 13.





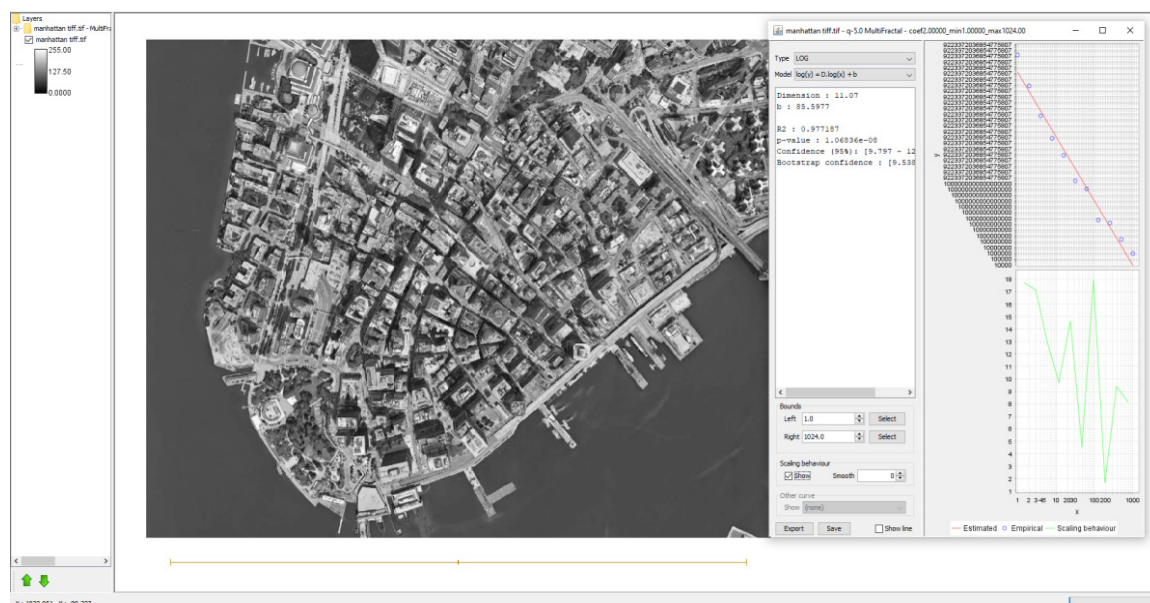


Figure 13. Results of the fractal analysis where: (a) Radial all points, (b) Box counting, and (c) Dilation. Source: the authors.

The application of this multi-index framework successfully decoded the complex urban structure of Lower Manhattan. The area was found to have high climate vulnerability and a complex, multi-nodal spatial configuration where economic core functions occupy the most accessible locations. The economic models project a short-term contraction and foster the retail sector and services. Pedestrian flow is concentrated and temporally variable. Ultimately, the district's identity is characterized by an organically evolved, highly complex fractal structure that supports its dense, mixed-use functionality see figure 14.

#### 4. LIMITATIONS

The authors addressed the challenges that reveal their current limits and point to a need for further development.

1. Data integration showed difficulty, as urban systems generate information at multiple scales and across different domains that do not always align. Translating mathematical abstractions into a practical urban design application also introduced tensions, since the theory requires a simplification when applied to policy contexts. Furthermore, the focus on the CBD, though justified by its economic centrality, fosters the need to test the model in peripheral zones and across metropolitan or regional networks where different dynamics may predominate.

2. This research highlights several epistemic and ethical considerations in its approach. Translating qualitative urban life into mathematical abstractions involves a reductive process. The proposed model captures configurational and economic dynamics, but downplays the importance of cultural meaning, historical context, and social equity concerns.

3. Urban designers should be careful not to let the apparent objectivity of the model's outputs replace community input and political discussion. The ethical implications of data sources, especially in urban computing, should also be recognized. While this study uses aggregated public data, future work that incorporates more detailed data must rigorously address consent and anonymity.

#### 5. CONCLUSION

This study introduced and tested a model that combines mathematical coding, graph theory, and urban structure theory to analyze the complexity of CBDs and their economic performance. The model goes beyond traditional methods that often limit themselves to descriptive mapping or narrowly defined economic indicators. It provides a framework that benefits from the precision of the sciences, including graph-based analysis and the contextual grounding of urban theory. It shows that urban structures can be understood as rule-based, connected graphs shaped by physical form, socio-economic processes, and vulnerability to potential hazards. The model's strength lies in its ability



to translate the abstract nature of mathematical coding into practical indices relevant for urban evaluation. Codes, laws, and sequences, such as Zipf's law, scaling, Kleiber's law, syntax, and game theory, were integrated into the model to identify structural patterns and predict potential outcomes in urban areas.

When incorporated into graph theory, these codes became relational, allowing nodes and paths to represent land uses, connections, and flows that influence life in the CBD. The model predicts shifts in land values, housing supply, and retail composition, while also considering hazards like flooding, heat, wind, and fire. Additionally, it offers a way to analyze the resilience and adaptability of CBDs as economic and spatial centers of cities.

The practical application to Lower Manhattan CBD confirmed the analytical depth of this approach. Vulnerability assessments showed that the district faces significant environmental risks, with storm surge and flooding presenting particularly acute threats. At the same time, the system displayed only moderate adaptive capacity despite its complexity. Graph-based algorithms revealed crucial nodes in the urban network, identifying points of strength as well as potential failure. Space syntax analysis highlighted layered accessibility patterns, showing that commercial and institutional spaces dominated connectivity while residential areas remained more isolated. These results revealed a duality: the CBD is both highly integrated and economically vibrant, yet fragile in the face of compounding hazards.

The broader implications of this study are significant. By demonstrating that urban structure can be analyzed as a codified and connected system, the research challenges the dominance of purely descriptive urban studies. It suggests that predictive, mathematically informed approaches have much to offer. For planners, the model can serve as a decision-support tool that connects economic performance with resilience, allowing for the anticipation of vulnerabilities and opportunities before they lead to crises. For policymakers, it shows that CBDs, as economic drivers, must be understood not only in terms of their productivity but also in relation to their fragility, especially under climate and environmental stressors. For the academic community, the model provides an example of interdisciplinary synthesis, bridging mathematics, spatial analysis, and urban theory in ways that encourage new inquiries.

Ultimately, this research confirms that the city, especially the CBD, cannot be adequately understood through a single-dimensional analysis. It is a dynamic entity shaped by layers of economic transactions, social interactions,

environmental pressures, and cultural practices, all organized by networks of accessibility and flows. The proposed model captures this complexity by integrating multiple perspectives into one framework, offering both explanatory and predictive power. While challenges remain regarding data, practical implementation, and broader applicability, this study establishes a foundation for future work.

In conclusion, this model represents an important step toward systematizing urban structure analysis. By bringing together mathematical coding, graph theory, and urban theory, it enhances both conceptual and practical understandings of how CBDs operate, how they perform economically, and how they respond to vulnerabilities. More importantly, it emphasizes the need for tools that reflect the complexity of cities themselves, moving beyond description to prediction and from static representation to dynamic modeling. This work lays the groundwork for planning approaches that are more resilient, adaptive, and capable of sustaining economic vitality amid uncertainty.

#### DATA AVAILABILITY

All data used in the analysis are cited in the body of the manuscript and listed in the references.

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